جامعة تكريت
كلية التربية للبنات
قسم الرياضيات

المرحلة : الرابعة
المادة : الاحصاء الرياضي

The Best Critical Region

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The Best Critical Region:-

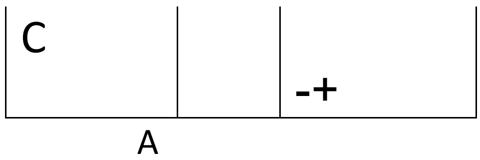
Let C denote a subset of sample space then C is called the b.c.r of size α for testing the simple hypothesis

$H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$

If for every subset A of the Sample Space for which

$p[\,(x_1, x_2 \dots., x_n) \in A; H_0] = \alpha$

1- $p[\,(* x_1, x_2 \dots., x_n) \in C; H_0] = \alpha$
2- $p[\,(x_1, x_2 \dots., x_n) \in C/H_1] \geq p[\,(x_1, x_2 \dots., x_n) \in A; H_1]$



$$p(I\ error)_C = \alpha$$

Ex .: Let x have

a binomial distribution with parameters n =10 and

$p \epsilon \left\{p; p = \frac{1}{4}, \frac{1}{2}\right\}$ the simple hypothesis $H_0 : p = \frac{1}{2}$ is rejected , and the alternative simple hypothesis $H_1 : p = \frac{1}{4}$ is accepted , if the observed value of x_1, a random sample of size 1, is less than or equal to 3. Find the power function of the test?

sol.

$$\alpha = \sum_{0}^{3} p\left(x; p = \frac{1}{2}\right)$$

$$= p(x=0) + p(x=1) + p(x=2) + p(x=3)$$

$$= C_0^{10}(\tfrac{1}{2})^0(\tfrac{1}{2})^{10} + C_1^{10}(\tfrac{1}{2})^1(\tfrac{1}{2})^9 + C_2^{10}(\tfrac{1}{2})^2(\tfrac{1}{2})^8 + C_3^{10}(\tfrac{1}{2})^3(\tfrac{1}{2})^7$$

p .o .T . $= \sum_0^3 p\left(x; p = \frac{1}{4}\right)$

$$= \quad C_0^{10}(\tfrac{1}{4})^0(\tfrac{3}{4})^{10} \; + C_1^{10}(\tfrac{1}{4})^1(\tfrac{3}{4})^9 \; + C_2^{10}(\tfrac{1}{4})^2(\tfrac{3}{4})^8 + \; C_3^{10}(\tfrac{1}{4})^3(\tfrac{3}{4})^7$$

$$\beta = \quad 1 - P.O.T.$$

Ex. Let $x_1, x_2, \ldots\ldots x_n$ be a random sample of size 10 from a Normal distribution $N\sim(0,\sigma^2)$, find a best Critical region of size $\alpha = 0.05$ for testing $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 = 2$ .Is the a best Critical region of size $\alpha = 0.05$ for testing $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 = 4$? Against $H_1 : \sigma^2 = \sigma_1^2 > 1$?

sol.

$$\frac{L(x_1,x_2,\ldots\ldots,x_n)\backslash H_0)}{L(x_1,x_2,\ldots\ldots,x_n)\backslash H_1)} \le k$$

$$f(x,\theta) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-1}{2}(\frac{x-\mu}{\sigma^2})^2}$$

$$= \frac{1}{\sqrt{2\pi\sqrt{2}}} e^{\frac{-1}{2}(\frac{x}{2})^2}$$

$$= \frac{\frac{1}{\sqrt{2\pi}}e^{\frac{-1}{2}x_1^2}\ldots\ldots\frac{1}{\sqrt{2\pi}}e^{\frac{-1}{2}x_n^2}}{\frac{1}{\sqrt{2\pi\sqrt{2}}}e^{\frac{-1}{2}(\frac{x}{2})^2}\ldots\ldots\frac{1}{\sqrt{2\pi\sqrt{2}}}e^{\frac{-1}{2}(\frac{x_n}{2})^2}} \qquad \le k$$

$$\frac{e^{-\frac{1}{2}\sum x_i^2}}{2^{\frac{n}{2}}e^{-\frac{1}{4}\sum x_i^2}} \le \quad k$$

$$\rightarrow \; e^{-\frac{1}{2}\sum x_i^2+\frac{1}{4}\sum x_i^2} \; \le k \,.\, 2^{\frac{n}{2}}$$

$$\rightarrow \; e^{-\frac{1}{4}\sum x_i^2} \; \le k \,.\, 2^{\frac{n}{2}}$$

$$\rightarrow \; -\frac{1}{4}\sum x_i^2 \; \le \ln k \,.\, 2^{\frac{n}{2}}$$

$$\rightarrow \sum x_i^2 \ge \; -4\ln k \,.\, 2^{\frac{n}{2}}$$

$\rightarrow \sum x_i^2 \ge \quad C$ is B.C.R

$$X_i\backslash\, H_0 \sim N(0,1)$$

$$X_i^2\backslash H_0 \sim x^2(1)$$

$$\sum x_i^2 \backslash H_0 \sim x^2(10)$$

$$p(\sum x_i^2 \geq C \backslash H_0) = 0.05$$

$$p(\sum x_{10}^2 \geq C \backslash H_0) = 0.05$$

$$p(\sum x_{10}^2 \geq C \backslash H_0) = 0.95$$

$\rightarrow \quad C = 18.3$

$H_0 : \sigma^2 = 1 \quad , \; H_1 : \sigma^2 = 4$

$$\frac{(\frac{1}{\sqrt{2\pi}})^n e^{\frac{-1}{2} x_i^2}}{(\frac{1}{\sqrt{2\pi}})^n e^{\frac{-1}{2} x_i^2}} \quad \leq k$$

$$\rightarrow e^{-\frac{1}{2}\sum x_i^2 + \frac{1}{3}\sum x_i^2} \quad \leq k \,.\, 2^n$$

$$\rightarrow -\frac{3}{8}\sum x_i^2 \leq \ln k \,.\, 2^n$$

$$\rightarrow \sum x_i^2 \geq C$$

∴ C= 18.3

EX. Let $x_1, x_2, \ldots \ldots x_n$ be a random sample from a distribution having p.d.f. of the form $f(x;\theta) = \theta x^{\theta - 1}$, $0 < x < 1$, $zero$ elsewhere, show that a best Critical region for testing $H_0 : \theta \; = 1$ against $H_1 : \theta = 2$, is C ={$(x_1, x_2, \ldots \ldots x_{n)}$; C $\leq \prod_{i=1}^{n} x_i$}

sol.

$$\frac{L(x_1, x_2, \ldots\ldots, x_n;\, \theta) \backslash H_0)}{L(x_1, x_2, \ldots\ldots, x_n; \theta) \backslash H_1)} \leq k$$

$$= \frac{1}{2^n \prod_{i=1}^{n} x_i} \leq k$$

$$\rightarrow 2^n \prod_{i=1}^{n} x_i \quad \geq \; k^*$$

$$\rightarrow \prod_{i=1}^{n} x_i \geq \frac{k^*}{2^n}$$

$\rightarrow \prod_{i=1}^{n} x_i \; \geq \; C$,Is the B.C.R.

Example : X is a random variable has a p.d.f of the form $f(x;\theta\ ) = \frac{1}{\theta}e^{\frac{x}{\theta}}\ ; x\ > 0$,it is desired to test the simple hypothesis $H_0 : \theta = 2$, against $H_1 : \theta = 4$ a random sample of size n=2 will be used , and the Critical region: $C = \{(\ x_1, x_2); 9.5\ \leq x_1 + x_2 \leq \infty\}$

sol.: تحت ظروف H_0

$$f(x_1;\theta_0) * f(x_2;\theta_0) = \frac{1}{2}e^{\frac{-x_1}{2}} * \frac{1}{2}e^{\frac{-x_2}{2}} \qquad 0\ < x_1\ < \infty$$

$$= \frac{1}{2}e^{-\frac{1}{2}(x_1+x_2)}$$

$$p\big((x_1, x_2)\epsilon C\big) = 1\ - P((x_1, x_2\epsilon\bar{C})$$

$$1 - \int_0^{9.5}\int_0^{9.5-x_1} f(x_1, x_2) d_{x_1}\, d_{x_2} = 0.05 = \alpha$$

تحت ظروف H_1

$$f(x_1;\theta_1) * f(x_2;\theta_1) = \frac{1}{4}e^{\frac{-x_1}{4}} * \frac{1}{4}e^{\frac{-x_2}{4}}$$

$$= \frac{1}{16}e^{\frac{-1}{4}(x_1+x_2)}$$

$$\beta = \int_0^{9.5}\int_0^{9.5-x_1} \frac{1}{16}e^{\frac{-1}{4}(x_1+x_2)} d_{x_1}\, d_{x_2}\ \ = 0.69$$

power of the test =1- β

$$1\text{-}\ \beta = 1 - 0.69 = 0.31$$

ومن الواضح أن قوة الاختبار قليلة والخطأ من النوع الاول