جامعة تكريت – كلية التربية للبنات –قسم الرياضيات

المرحلة : الرابعة

المادة: التبولوجيا العامة

عنوان المحاضرة : استمرارية الفضاء التبولوجية

مدرس المادة : أ .د. رنا بهجت ياسين

الايميل الجامعي : Zain 2016@ tu.edu.iq

Definition (1)

Let (M,τ) and (N,σ)be a topological spaces. A map $f:(M,\tau)\to(N,\sigma)$is continuous if $\mathfrak{f}^{-1}(\mathbb{W})$ of each open set $\mathbb{W}$ in $\breve{\mathcal{M}}$is open set in $\mathcal{M}$.

Proposition (2)

Let (M,τ), $(N,\sigma)$and $\left(\bar{\bar{\mathcal{M}}},\Im\left(\bar{\bar{\mathbb{X}}}\right)\right)$ are three topological spaces. If $\mathfrak{f}:(M,\tau)\to(N,\sigma)$ and g: $(N,\sigma)\to\left(\bar{\bar{\mathcal{M}}},\Im\left(\bar{\bar{\mathbb{X}}}\right)\right)$ are continuous maps, then go$f$: $(M,\tau)\to\left(\bar{\bar{\mathcal{M}}},\Im\left(\bar{\bar{\mathbb{X}}}\right)\right)$ is continuous.

Proof.

Suppose that $\mathbb{H}$ is open set in $\left(\bar{\bar{\mathcal{M}}},\Im\left(\bar{\bar{\mathbb{X}}}\right)\right)$. Since g is continuous, whenever $g^{-1}(\mathbb{H})$ is open set in (N,σ). Now, $(g\circ\mathfrak{f})^{-1}\mathbb{H}=\mathfrak{f}^{-1}(g^{-1}(\mathbb{H}))$.

So $\mathfrak{f}^{-1}(\mathbb{H})$is open set in (M,τ)because $g^{-1}(\mathbb{H})$ is open set in (N,σ)). Since $\mathfrak{f}$ is continuous map. Then go$\mathfrak{f}$: $(M,\tau)\to\left(\bar{\bar{\mathcal{M}}},\Im\left(\bar{\bar{\mathbb{X}}}\right)\right)$ is continuous.

The study present the definition and characterization of continuous map in terms of inverse image of closed.

Theorem (3)

Let $\mathbb{W}$ be *closed* set in $(N,\sigma)$and let $f:(M,\tau)\to(N,\sigma)$ is continuous map if and only if $\mathfrak{f}^{-1}(\mathbb{W})$ is closed set in $\mathcal{M}$,for each closed set $\mathbb{W}$ in (N,σ)

Proof.

Suppose that $f:(M,\tau)\to(N,\sigma)$ is continuous and $\mathbb{W}$ is closed set in (N,σ)we get $\mathbb{W}^c$ is *open* set in (N,σ).Since $\mathfrak{f}^{-1}(\mathbb{W}^c)=\mathfrak{f}^{-1}(\mathbb{W})^{c}$ is $\mathcal{N}_{\mathcal{P}}$_Oset in (M,τ) hence $\mathfrak{f}^{-1}(\mathbb{W})$ is closed set in (M,τ) whenever $\mathbb{W}$ is closed set in(N,σ).

Conversely,

For each $\mathbb{W}$ is closed set in $(N,\sigma)$and $\mathfrak{f}^{-1}(\mathbb{W})$ is closed set in (M,τ) .

Suppose that(M,τ) is open set in (N,σ)then $\mathcal{W}^{c}$ is closed set in (N,σ).

By hypothesis $\mathfrak{f}^{-1}(\mathcal{W}^{c}) = \mathfrak{f}^{-1}(\mathcal{W})^{c}$ is closed set in (M,τ) , hence $\mathfrak{f}^{-1}(\mathcal{W})$is open set in (M,τ) . We get $\mathfrak{f}$ is continuous.

In the following proposition, we establish a characterization of continuous maps in terms of closure.

Proposition (4)

A map $f:(M,\tau) \to (N,\sigma)$is continuous if and only if $\mathfrak{f}(\mathrm{cl}(\mathbb{A})) \subseteq \mathrm{cl}(\mathfrak{f}(\mathbb{A}))$ for each $\mathbb{A} \subseteq \mathcal{M}$.

Proof.

Let $f:(M,\tau) \to (N,\sigma)$ be a continuous map and $\mathbb{A} \subseteq \mathcal{M}$, be arbitrary ,so $\mathfrak{f}(\mathbb{A}) \subseteq$ N.Then $\mathrm{cl}(\mathfrak{f}(\mathbb{A}))$ is closed in (N,σ). By theorem (3), we get $\mathfrak{f}^{-1}(\mathrm{cl}(\mathfrak{f}(\mathbb{A})))$ is closed in (M,τ) . Since $\mathfrak{f}(\mathbb{A}) \subseteq \mathrm{cl}(\mathfrak{f}(\mathbb{A})) \to \mathbb{A} \subseteq \mathfrak{f}^{-1}(\mathrm{cl}\ (\mathfrak{f}(\mathbb{A})))$ and $\mathrm{cl}\big(\mathbb{A}) \subseteq \mathrm{cl}(\mathfrak{f}^{-1}(\mathrm{cl}\ (\mathfrak{f}(\mathbb{A}))\big) =$

$\mathfrak{f}^{-1}(\mathrm{cl}(f(\mathbb{A}))$. Then $\mathfrak{f}(\mathrm{cl}(\mathbb{A})) \subseteq \mathrm{cl}(\mathfrak{f}(\mathbb{A}))$ for each $\mathbb{A} \subseteq \mathcal{M}$.

Conversely,

For each $\mathbb{A} \subseteq \mathcal{M}$ which that, $\mathfrak{f}(\mathrm{cl}(\mathbb{A})) \subseteq \mathrm{cl}(\mathfrak{f}(\mathbb{A}))$. If $\mathbb{D}$ is closed set in (N,σ). Since $\mathrm{cl}(\mathbb{D}) = \mathbb{D}$ then $\mathfrak{f}(\mathrm{cl}\big(\mathfrak{f}^{-1}(\mathbb{D})\big)) \subseteq \mathrm{cl}(\mathfrak{f}\big(\mathfrak{f}^{-1}(\mathbb{D})\big)$ which implies that $cl\big(\mathfrak{f}^{-1}(\mathbb{D})\big) \subseteq \mathbb{D}$.

But, always $\mathfrak{f}^{-1}(\mathbb{D}) \subseteq \mathrm{cl}\big(\mathfrak{f}^{-1}(\mathbb{D})\big)$, so that $\mathrm{cl}\big(\mathfrak{f}^{-1}(\mathbb{D})\big) = \mathfrak{f}^{-1}(\mathbb{D})$ we get $\mathfrak{f}^{-1}(\mathbb{D})$ is closed set in (N,σ). Then $\mathfrak{f}$ is continuous map by using (theorem 3).

Now we present the definition and characterization of Nano Penta totally continuous map.

Definition

A map $f:(M,\tau)\rightarrow(N,\sigma)$is totally continuous if $\mathfrak{f}^{-1}(\mathbb{W})$ of each open set open subset $\mathbb{W}$ in $\breve{\mathcal{M}}$is $clopen$ set in $\mathcal{M}$.

Theorem

Let $f:(M,\tau)\rightarrow(N,\sigma)$be totally continuous . Then $\mathfrak{f}$ is con map.

Proof.

To prove this we to use Theorem above ■

Theorem (4.1.15)

A map $f:(M,\tau)\rightarrow(N,\sigma)$is totally continuous iff $\mathfrak{f}^{-1}(\mathbb{W})$ of each $\mathbb{W}$ closed set in N is $clopen$ in $\mathcal{M}$.

Proof.

If $f:(M,\tau)\rightarrow(N,\sigma)$is a totally continuous map and $\mathbb{W}$ is closed set in N ,we get $\mathbb{W}^c$ is closed set in N.

Then $\mathfrak{f}^{-1}(\mathbb{W}^c)$ is clopen in $\mathcal{M}$(because $\mathfrak{f}$ is totally continuous thus $\mathfrak{f}^{-1}(\mathbb{W})$ is clopen in $\mathcal{M}$.

Conversely,

Suppose that $\mathfrak{f}^{-1}(\mathbb{W})$ is clopen in $\mathcal{M}$, for each $\mathbb{W}$ is closed set in N and let $\mathbb{W}^c$ is open set in N, according to the hypothesis $(\mathfrak{f}^{-1}(\mathbb{W}))^c= \mathfrak{f}^{-1}(\mathbb{W}^c)$ is closed set in $\mathcal{M}$.

Then $\mathfrak{f}:\mathcal{M}\rightarrow$ N is a totally continuous (because $\mathfrak{f}^{-1}(\mathbb{W})$ is $clopen$ in $\mathcal{M}$). ■

Theorem

Let $f:(M,\tau)\to(N,\sigma)$ be map. If ƒ is totally continuous , then ƒ is Con.

Proof.

Suppose that $f:(M,\tau)\to(N,\sigma)$is a totally continuous map, by using(Proposition above)

we get $ƒ^{-1}(\mathbb{W})$ of each closed set. $\mathbb{W}$ in N, is clopen in $\mathcal{M}$.

Since every clopen in $\mathcal{M}$is closed set in $\mathcal{M}$ become $ƒ^{-1}(\mathbb{W})$ of each $\mathbb{W}$ is closed set in N ,is closed set in $\mathcal{M}$.

Thus $f:(M,\tau)\to(N,\sigma)$ is a Con. map. ■