Ninth lecture of Integration Methods

New method

(19) Nineteenth method: Integration of inverse trigonometric functions

We can integrate inverse trigonometric functions by use the formula

$$\int f^{-1}(x)dx = xf^{-1}(x) - \int f(y)dy \quad \text{where} \quad y = f^{-1}(x)$$

Example (65): Evaluate $I = \int \tan^{-1} x dx$

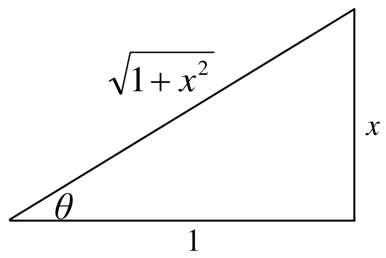
$$I = \int \tan^{-1} x dx = x\tan^{-1} x - \int \tan y dy \;, \text{ where } \; y = \tan^{-1}(x)$$

$$I = x\tan^{-1} x - \int \frac{\sin y}{\cos y} dy = x\tan^{-1} x + \int \frac{-\sin y}{\cos y} dy$$

$$I = x\tan^{-1} x + \ln|\cos y| + c$$

$$I = x\tan^{-1} x + \ln|\cos(\tan^{-1} x)| + c$$

let $\theta = \tan^{-1} x$. and draw a reference triangle with the side opposite of θ having length x and the side adjacent to θ having length 1 in order to get $\tan\theta = x$.



$$I = x\tan^{-1} x + \ln|\cos\theta| + c$$

$$I = x\tan^{-1} x + \ln\left|\frac{1}{\sqrt{1+x^2}}\right| + c = x\tan^{-1} x + \ln|(1+x^2)^{-1/2}| + c$$

$$I = x\tan^{-1} x - \frac{1}{2}\ln(1+x^2) + c$$

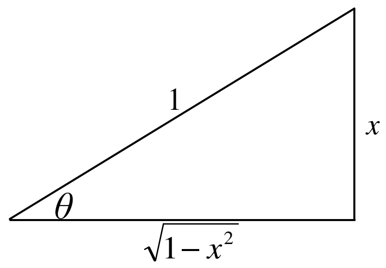
Example (66): Evaluate $I = \int \sin^{-1} x dx$

$$I = \int \sin^{-1} x dx = x\sin^{-1} x - \int \sin y dy \;, \text{ where } \; y = \sin^{-1}(x)$$

$$I = x\sin^{-1} x + \cos y + c$$

$$I = x\sin^{-1} x + \cos(\sin^{-1} x) + c$$

let $\theta = \sin^{-1} x$. and draw a reference triangle with the side opposite of θ having length $\sqrt{1-x^2}$ and the side adjacent to θ having length x in order to get $\sin\theta = x$.



$$I = x\sin^{-1} x + \cos\theta + c$$

$$I = x\sin^{-1} x + \sqrt{1-x^2} + c$$

In the same way we can integrate the others inverse trigonometric functions.