Tenth lecture of Integration Methods

New method

(20) Twentieth method: clever rearrangement method

In some Integrals not apparent that the chain rule is involved therefore we need clever rearrangement in order to the chain rule is involved.

Example (67): Evaluate $I=\int x^3\sqrt{1-x^2}\,dx$

There are two factors in this expression, x^3 and $\sqrt{1-x^2}$ but it is not apparent that the chain rule is involved. Some clever rearrangement reveals that it is:

$$I=\int x^3\sqrt{1-x^2}\,dx=\int x.x^2\sqrt{1-x^2}\,dx$$

$$I=\int x(1-1+x^2)\sqrt{1-x^2}\,dx$$

$$I=\int x[1-(1-x^2)]\sqrt{1-x^2}\,dx$$

$$I=\int x[\sqrt{1-x^2}-(1-x^2)^{3/2}]dx$$

$$I=\int x\sqrt{1-x^2}\,dx-\int x(1-x^2)^{3/2}\,dx$$

$$I=\frac{-1}{2}\int(1-x^2)^{1/2}(-2x)dx+\frac{1}{2}\int(1-x^2)^{3/2}(-2x)dx$$

$$I=\frac{-1}{2}.\frac{(1-x^2)^{3/2}}{3/2}+\frac{1}{2}\frac{(1-x^2)^{5/2}}{5/2}+c$$

$$I=\frac{-1}{3}(1-x^2)^{3/2}+\frac{1}{5}(1-x^2)^{5/2}+c$$

Example (68): Evaluate $\int x^5\sqrt{2+x^3}\,dx$

$$I=\int x^5\sqrt{2+x^3}\,dx=\int x^2x^3\sqrt{2+x^3}\,dx$$

$$I=\int x^2[-2+(2+x^3)]\sqrt{2+x^3}\,dx$$

$$I=\int x^2[-2(2+x^3)^{1/2}+(2+x^3)^{3/2}]dx$$

$$I=-2\int(2+x^3)^{1/2}x^2dx+\int(2+x^3)^{3/2}x^2dx$$

$$I=\frac{-2}{3}\int(2+x^3)^{1/2}(3x^2)dx+\frac{1}{3}\int(2+x^3)^{3/2}(3x^2)dx$$

$$I=\frac{-2}{3}\cdot\frac{2}{3}(2+x^3)^{3/2}+\frac{1}{3}\cdot\frac{2}{5}(2+x^3)^{5/2}+c$$

$$I=\frac{-4}{9}(2+x^3)^{3/2}+\frac{2}{15}(2+x^3)^{5/2}+c$$