جامعة تكريت

كلية التربية للبنات

قسم الرياضيات

المرحلة الثانية

مادة التفاضل المتقدم

الرسم في الاحداثيات القطبية Graphing in Polar Coordinates

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- **(1) Lines in Polar Coordinates**

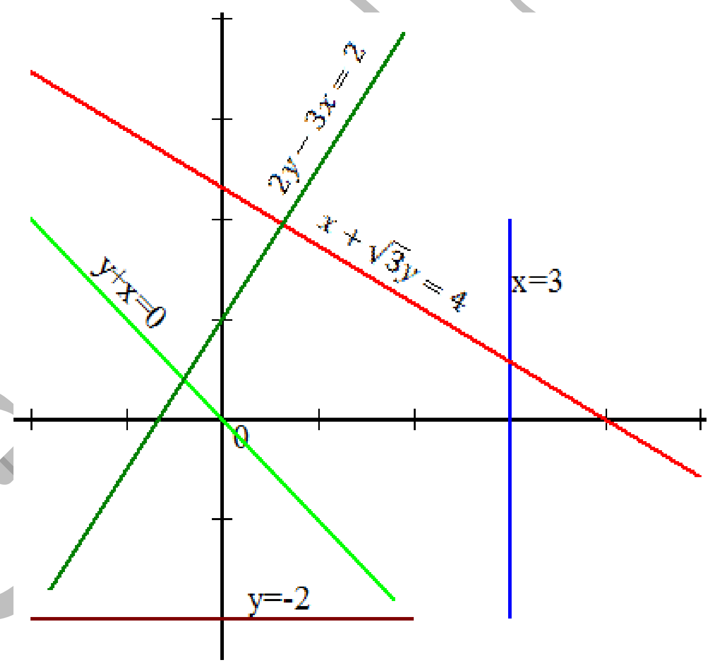
$$ax + by = c \quad \longrightarrow \quad r(a\cos\theta + b\sin\theta) = c \qquad a, b, c \in R$$

Ex: Sketch the following in polar coordinates

1) $r\cos\theta = 2$ 2) $r = 3\sec\theta$ 3) $r = 2\sec\theta$ 4) $r = -2\csc\theta$

5) $\theta = \frac{3\pi}{4}$ 6) $r = \frac{2}{2\sin\theta - 3\cos\theta}$.

Sol:



1) $r\cos\theta = 2 \longrightarrow x = 2.$
2) $r = 3\sec\theta \Rightarrow r\cos\theta = 3$
$\longrightarrow x = 3.$
3) $r = 2\sec\theta \Rightarrow r\cos\left(\theta - \frac{\pi}{3}\right) = 2$

$$\Rightarrow r\left(\cos\theta\cos\left(\frac{\pi}{3}\right) + \sin\theta\sin\left(\frac{\pi}{3}\right)\right) = 2$$

$$\Rightarrow r\left(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right) = 2$$

$$\Rightarrow r\cos\theta + \sqrt{3}\sin\theta = 4$$

$$\longrightarrow x + \sqrt{3}y = 4.$$

4) $r = -2\csc\theta \Rightarrow r\sin\theta = -2 \longrightarrow y = -2.$
5) $\theta = \frac{3\pi}{4}$, since $\theta = \tan^{-1}\left(\frac{y}{x}\right)$,

so $\tan^{-1}\left(\frac{y}{x}\right) = \frac{3\pi}{4} \Rightarrow \frac{y}{x} = \tan\left(\frac{3\pi}{4}\right)$

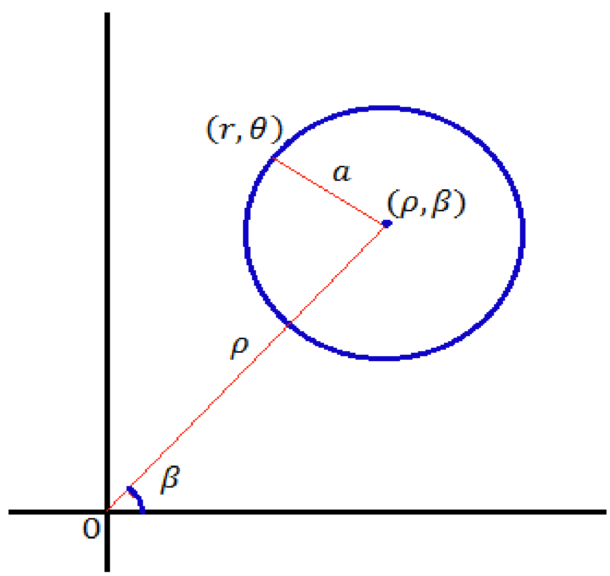
$\Rightarrow \frac{y}{x} = -1 \Rightarrow y + x = 0.$

6) $r = \frac{2}{2\sin\theta - 3\cos\theta} \Rightarrow r(2\sin\theta - 3\cos\theta) = 2$
$\longrightarrow 2y - 3x = 2.$

(2) Circles in Polar Coordinates

$(x-h)^2+(y-k)^2=a^2 \quad \longrightarrow \quad r^2=a^2-\rho^2+2r\rho\cos(\theta-\beta)$

$a^2-\rho^2=r^2-2r\rho\cos(\theta-\beta)$; where



ρ:بعد المركز عن نقطة الاصل

a : نصف قطر الدائرة

(r,θ): أحداثيات نقطة على محيط الدائرة

β : زاوية التدوير

- Spatial case:

1) When $\rho=a$, $r^2=2r\rho\cos(\theta-\beta)$ or $r=2\rho\cos(\theta-\beta)$, so if $\beta=0$, $r=2\rho\cos\theta$also if $\beta=\frac{\pi}{2}$, $r=2\rho\sin\theta$.
2) If $\rho=0$, we have $r=a$.

Ex: Sketch the following in polar coordinates: (H.W.)

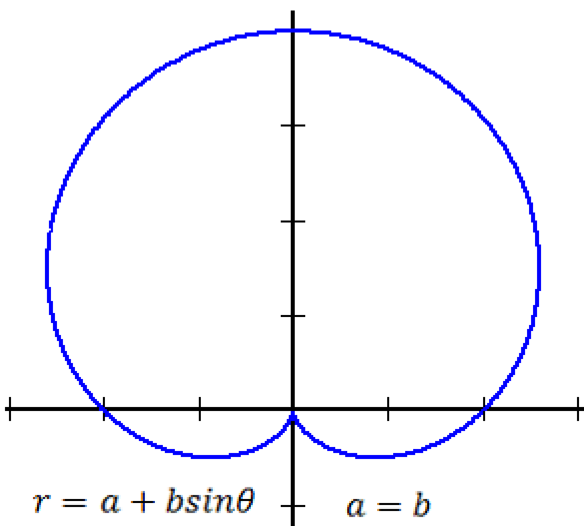
1) $r=4\cos\theta$ 2) $r=-4\cos\theta$ 3) $r=5\sin\theta$ 4) $r^2=9$.

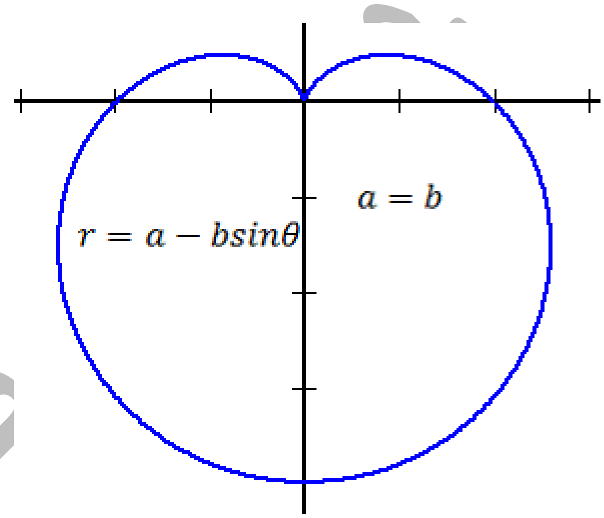
(3) Cardioid

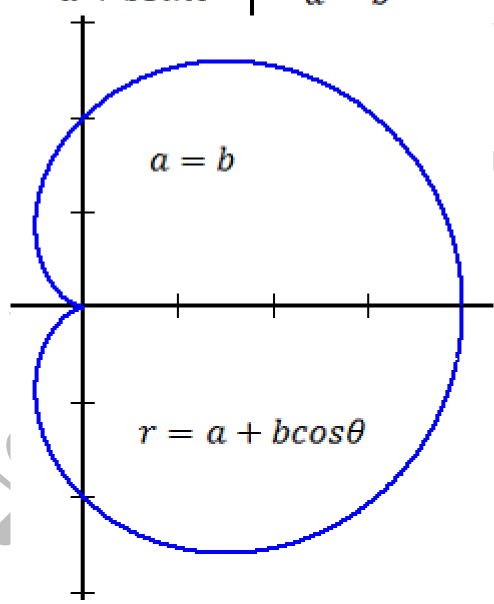
توجد أربعة احتمالات لشكل (Lima cons) وذلك نسبتاً للمقدار $\frac{a}{b}$.

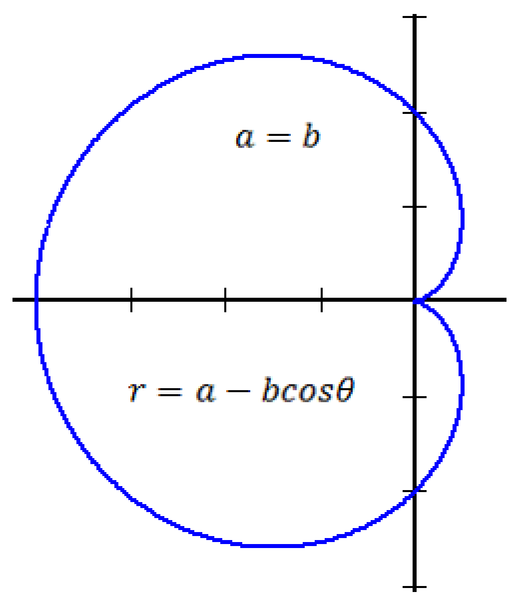
$$\left[\frac{a}{b} < 1; \frac{a}{b} = 1; 1 < \frac{a}{b} < 2; \frac{a}{b} \geq 2\right]$$

1) when $a = b$.









Ex: Sketch the following graphs:

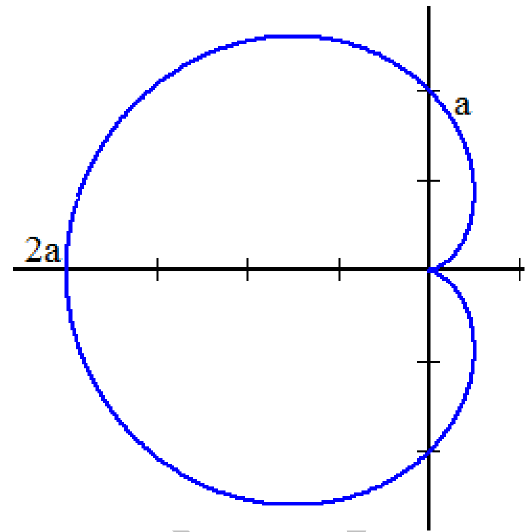
1) $r = a(1 - cos\theta)$ 2) $r = a(1 + cos\theta)$

3) $r = a(1 + sin\theta)$4) $r = a(1 - sin\theta)$

Sol: 1) $r = a(1 - cos\theta)$,

$0 \le \theta \le \pi$, since the curve

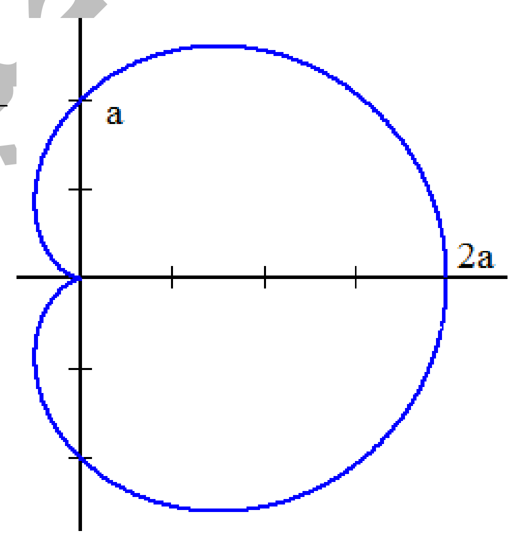
symmetric about X-axis.



θ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
r	0	$\frac{a}{2}$	a	$\frac{3a}{2}$	$2a$

2) $= a(1 + cos\theta)$, $0 \le \theta \le \pi$, since the curve symmetric about X-axis

θ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
r	$2a$	$\frac{3a}{2}$	a	$\frac{a}{2}$	0



3) & 4) (H.W.)

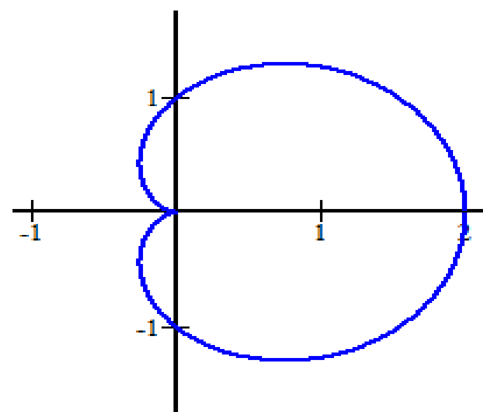
Ex: Sketch the following:

1) $r = 1 + cos\theta$ 2) $r = 2 + cos\theta$ 3) $r = 1 + 2cos\theta$

4) $r = 2 - cos\theta$ 5) $r = 1 - 2cos\theta$

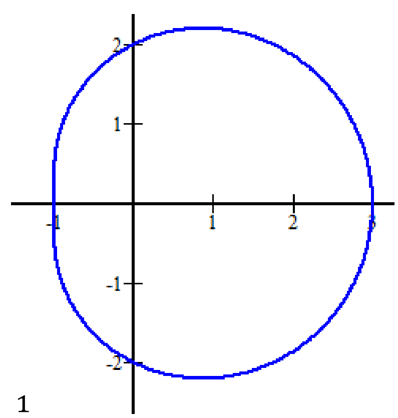
sol: 1) $= 1 + cos\theta$, symmetric about X-axis and $a = b$

θ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
r	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0



(2) $r = 2 + cos\theta$, then $\frac{a}{b} = 2$

θ	0	$\frac{\pi}{2}$	π
r	3	2	1



(3) $r = 1 + 2cos\theta$, Symmetric about X-axis $\frac{a}{b} = \frac{1}{2}$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
r	3	2.7	2	1	0	-1