جامعة تكريت

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# بعض انواع الاختبارات على المتسلسلات غير المنتهية

# Some types of test for Infinite Series

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- ## (4) The Comparison Test

**Theorem:** Let $\sum a_n$ , $\sum C_n$ and $\sum d_n$ be series with non-negative terms. Suppose that for some integer N. $d_n \le a_n \le C_n$ , $\forall n > N$.

1) If $\sum C_n$ converges, then $\sum a_n$ also converges.
2) If $\sum d_n$ diverges, then $\sum a_n$ also diverges.

**Ex:** Determine whether the series $\sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3}$ is converges or diverges ?

Sol: Choose $\sum_{n=1}^{\infty} \frac{5}{2n^2} = \sum C_n$ , $\sum C_n = \frac{5}{2} \sum_{n=1}^{\infty} \frac{1}{n^2}$ is Conv. {p-series p=2} and

$\sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3} < \sum_{n=1}^{\infty} \frac{5}{2n^2}$ $\forall n$,then by comparison test $\sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3}$ is Conv.

**Ex:** Does the series $\sum_{n=1}^{\infty} \frac{n+1}{n^2}$ Conv. or Div.?

Sol: Choose$\sum b_n$=$\sum_{n=1}^{\infty} \frac{1}{n}$ is Div. {p=1} when $a_1 = \frac{2}{1}$ , $b_1 = \frac{1}{1}$, $a_2 = \frac{3}{4}$ , $b_2 = \frac{1}{2}$ , $a_3 = \frac{4}{9}$ , $b_3 = \frac{1}{3}$. Then $a_n \ge b_n$ $\forall n$ . the series is divergent.

**Ex:** Does the series $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}-1}$ Conv. or Div.? H.W

- ## (5) The Ratio test

**Theorem:** (Ratio test)

Let $\sum a_n$ be a series with positive terms and suppose that

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = p \,, \quad \text{then}$$

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1) The series converges if $P < 1$.

2) The series diverges if $p > 1$ or $p$ is infinite.

3) The test is inconclusive if $p = 1$.

**Ex:** Investigate the convergence of the following series.

1) $\sum_{n=0}^{\infty} \frac{2^n+5}{3^n}$ 2) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ 3) $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$.

Sol: 1) $\frac{a_{n+1}}{a_n} = \frac{2^{n+1}+5}{3^{n+1}} \cdot \frac{3^n}{2^n+5} = \frac{1}{3} \cdot \frac{2^{n+1}+5}{2^n+5} = \frac{1}{3} \cdot \left(\frac{5 \cdot 2^{-n}+2}{5 \cdot 2^{-n}+1}\right) \to \frac{1}{3} \cdot 2 = \frac{2}{3}$. p<1 Conv.

2) $\frac{a_{n+1}}{a_n} = \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n} = \frac{(n+1)(n+1)^n}{(n+1)n!} \cdot \frac{n!}{n^n}$

$= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e > 1$. Div.

3) $\frac{a_{n+1}}{a_n} = \frac{(2n+2)!}{(n+1)!(n+1)!} \cdot \frac{n!n!}{(2n)!} = \frac{(2n+2)(2n+1)(2n)!}{(n+1)n!(n+1)n!} \cdot \frac{n!n!}{(2n)!}$

$= \frac{(2n+2)(2n+1)}{(n+1)(n+1)} = \frac{4n+2}{n+1} \to 4 > 1$. Div.

Ex: Test of the following if its convergence or diverges series.

(1) $\frac{1}{1.2} - \frac{1}{2.2^2} + \frac{1}{3.2^3} - \frac{1}{4.2^4} + \ldots$

يمكن كتابة المتسلسلة بالشكل التالي

$$\sum \frac{(-1)^{n+1}}{n.2^n}$$

$$p = \lim_{n\to\infty} \frac{an+1}{an}$$

$$p = \lim_{n\to\infty} \frac{(-1)^{n+2}}{(n+1)2^{n+1}} \cdot \frac{n.2^n}{(-1)^{n+1}}$$

$$p=\lim_{n\to\infty}\frac{(-1)^n.(-1)^2}{(n+1)2^n.2^1}.\frac{n.2^n}{(-1)^n(-1)^1}=\frac{-n}{2(n+1)}=\frac{-1}{2}\lim_{n\to\infty}\frac{n}{n+1}=\frac{-1}{2}\prec 1$$

Is conv.

$$(2)\sum_{n=1}^{\infty}\frac{4^n n!n!}{(2n)!}$$

$$p=\lim_{n\to\infty}\frac{an+1}{an}$$

$$p=\lim_{n\to\infty}\frac{4^{n+1}(n+1)!(n+1)!}{2(n+1)!}.\frac{2n!}{4^n n!n!}$$

$$p=\lim_{n\to\infty}\frac{4^n4(n+1)n!(n+1)n!}{(2n+2)(2n+1)2n!}.\frac{2n!}{4^n n!n!}=\frac{4(n+1)(n+1)}{(2n+2)(2n+1)}$$

$$p=4\lim_{n\to\infty}\frac{(\frac{n}{n}+\frac{1}{n})(\frac{n}{n}+\frac{1}{n})}{(\frac{2n}{n}+\frac{2}{n})(\frac{2n}{n}+\frac{1}{n})}=4\frac{(1+0)(1+0)}{(2+0)(2+0)}=4\frac{1}{4}=1$$

$$p=1$$

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$$\lim_{n\to\infty}an=1\neq 0$$

اذن المتسلسلة متباعدة.

**Ex:** Investigate the convergence of the following series. H.W

1) $\sum_{n=1}^{\infty}\frac{n!n!}{(2n)!}$ 2) $\sum_{n=1}^{\infty}\frac{2^n+5^n}{3^n}$ .

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## • (6) The Root Test

**Theorem:** (Root test)

Let $\sum a_n$ be a series with $a_n \geq 0$for $n \geq N$, and suppose that

$$\lim_{n\to\infty} \sqrt[n]{a_n} = p \;, \;\; \text{then}$$

(1) The series converges if $P < 1$.

(2) The series diverges if $p > 1$ or $p$ is infinite.

(3) The test is inconclusive if $p = 1$.

**Ex:** Test the series

1) $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$  2) $\sum_{n=1}^{\infty} \left(\frac{n+2}{n+4}\right)^n$  3) $\sum_{n=1}^{\infty} \left(\frac{1}{n+1}\right)^n$.

Sol: 1) $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converges, because $\sqrt[n]{\frac{n^2}{2^n}} = \frac{\left(\sqrt[n]{n}\right)^2}{2} \to \frac{1}{2} < 1$. Conv.

2) $\lim_{n\to\infty} \left[\left(\frac{n+2}{n+4}\right)^n\right]^{\frac{1}{n}} = \lim_{n\to\infty} \frac{n+2}{n+4} = 1$. The test is fails.

**Ex:** Find the values of $x$, that make $\sum_{n=1}^{\infty} \left(\frac{x^2+1}{3}\right)^n$ convergent.

Sol: By Root test $\lim_{n\to\infty} \left[\left(\frac{x^2+1}{3}\right)^n\right]^{\frac{1}{n}} = \lim_{n\to\infty} \frac{x^2+1}{3} < 1$.

$\therefore \frac{x^2+1}{3} < 1 \;\Rightarrow\; x^2 < 2 \;\Rightarrow\; -\sqrt{2} < x < \sqrt{2}$ .

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**Ex:** For what values of $x$ does the following series converges $\sum_{n=1}^{\infty} \frac{x^{2n+3}}{n^2}$ .

Sol: By Ratio test $\lim_{n\to\infty} \frac{x^{2n+5}}{(n+1)^2} \cdot \frac{n^2}{x^{2n+3}} = \lim_{n\to\infty} x^2 \cdot \frac{n^2}{n^2+2n+1} < 1.$

$\therefore\ x^2 < 1 \Rightarrow -1 < x < 1.$

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