

جامعة تكريت  
كلية التربية للبنات  
قسم الرياضيات

محاضرة بعنوان  
( التماثل )

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## Isomorphism - التماثل

Def \ let  $f: (R, +, \cdot) \rightarrow (R', \bar{+}, \bar{\cdot})$  be a ring homo. We said that  $f$  is isomorphism if  $f$  is

One - to - one and on to that is ; the function  $f: (R, +, \cdot) \rightarrow (R', \bar{+}, \bar{\cdot})$  is said isomorphism if satisfies these conditions :

1)  $f$  is homo

2)  $f$  is I-I

3)  $f$  is on to

Then its called  $(R, +, \cdot)$  isomorphism to  $(R', \bar{+}, \bar{\cdot})$ .

We we written  $(R, +, \cdot) \approx (R', \bar{+}, \bar{\cdot})$

Ex \ Let  $f: R \rightarrow R$  such that  $f(x) = 2^x - \forall x \in R$  show that  $f$  is a ring homo or not ?

sol \ 1- let  $a, b \in R$

$$f(a+b) = 2^{a+b} = 2^a \cdot 2^b \neq f(a) \cdot f(b) \text{ since } 1, 2 \in R$$

$$f(1, 2) = f(3) = 2^3 = 8$$

$$f(1) + f(2) = 2 + 2^2 = 2 + 4 = 6 \text{ } f \text{ is not hom.}$$

$f$  is not

$$2) \text{ let } a, b \in R \ni f(a) = f(b) \quad 2^a = 2^b$$

$$\ln 2^a = \ln 2^b$$

$$\frac{a \ln 2}{\ln 2} = \frac{b \ln 2}{\ln 2} \quad a = b$$

3) let  $2^a \in \mathbb{R} \rightarrow a \in \mathbb{R} \ni f(a) = 2^a$

Ex) let  $f: \mathbb{Z} \rightarrow \mathbb{Z} \ni f(a) = 2a \in \mathbb{Z}$  show that  $\mathbb{Z} \cong \mathbb{Z}$  or not?

Sol) 1- let  $a, b \in \mathbb{Z}$

$$F(a+b) = 2(a+b) = 2a + 2b = f(a) + f(b)$$

$$F(a \cdot b) = 2(a \cdot b) = 2a \cdot b \neq f(a) \cdot f(b) \text{ is not hom.}$$

Ex) let  $f: \mathbb{Z} \rightarrow \mathbb{Z} \ni f(a) = 2a \forall a \in \mathbb{Z}$  show that  $\mathbb{Z} \cong \mathbb{Z}$  or not?

$$F(2,3) = 2 \cdot 6 = 12$$

$$F(2) \cdot f(3) = 4 \cdot 6 = 24$$

$F$  is not homo.  $f$  is not iso

**Example :** Let  $R$  be a set of real number and  $"."$

**is a usual multiplication on  $R$ . show that  $(R/\{0\})$  is an abelian group**

**Sol/1- $R/\{0\} \neq \emptyset$  is a non-empty set**

2-Let  $a, b \in R/\{0\}$

→  $a \neq 0, b \neq 0$  then  $a.b \neq 0$

→  $a.b \in R/\{0\}$

$R/\{0\}$  is closed under  $"."$

i.e  $\forall a, b \in R/\{0\} \Rightarrow a.b \in R/\{0\}$  such that

$$(a.b) = a.b$$

$$, \forall a, b \in R/\{0\}$$

so  $"."$  is a binary operation on  $R/\{0\}$

3-Let  $a, b, c \in R/\{0\}$  then

$$a.(b.c) = (a.b).c$$

→  $(R/\{0\})$  is semi-group