جامعة تكريت
كلية التربية للبنات
قسم الرياضيات

محاضرة بعنوان
(التشاكل الطبيعي ومبرهنة التشاكل الاساسية)

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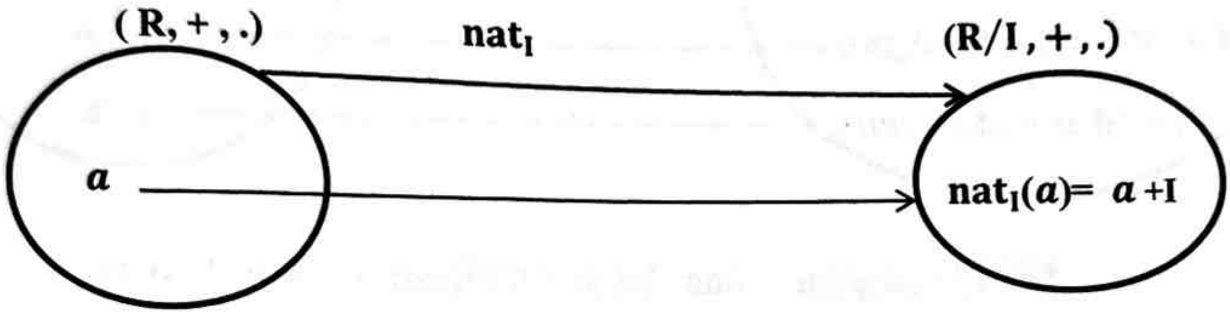
Definition :- (**The Natural Mapping**) دالة التطبيق الطبيعي

Let (R , + , .) be a ring and (I , + , .) be an ideal of a ring R . Then the **natural mapping** is function denoted by **nat_I** and is defined by :

$nat_I : (R, +, .) \rightarrow (R/I, +, .)$ such that, $nat_I(a) = a + I, \forall a \in R$.



Example : - (1) $f : (Z, +, .) \rightarrow (Z/(3), +, .)$ **is natural mapping**

(2) $g: (Z_8, +_8, \cdot_8) \rightarrow (Z_8/(\overline{4}), +, .)$ **is natural mapping**

Theorem (3-11):-

The natural mapping $nat_I : (R , + , .) \rightarrow (R/I, + , .)$ is ring homomorphism , on to function and the Kernel of natural mapping is equal to I .

Proof :- المعطى

$nat_I : (R, +, .) \rightarrow (R/I, +, .)$ S.t $nat_I(a) = a + I, \forall a \in R$

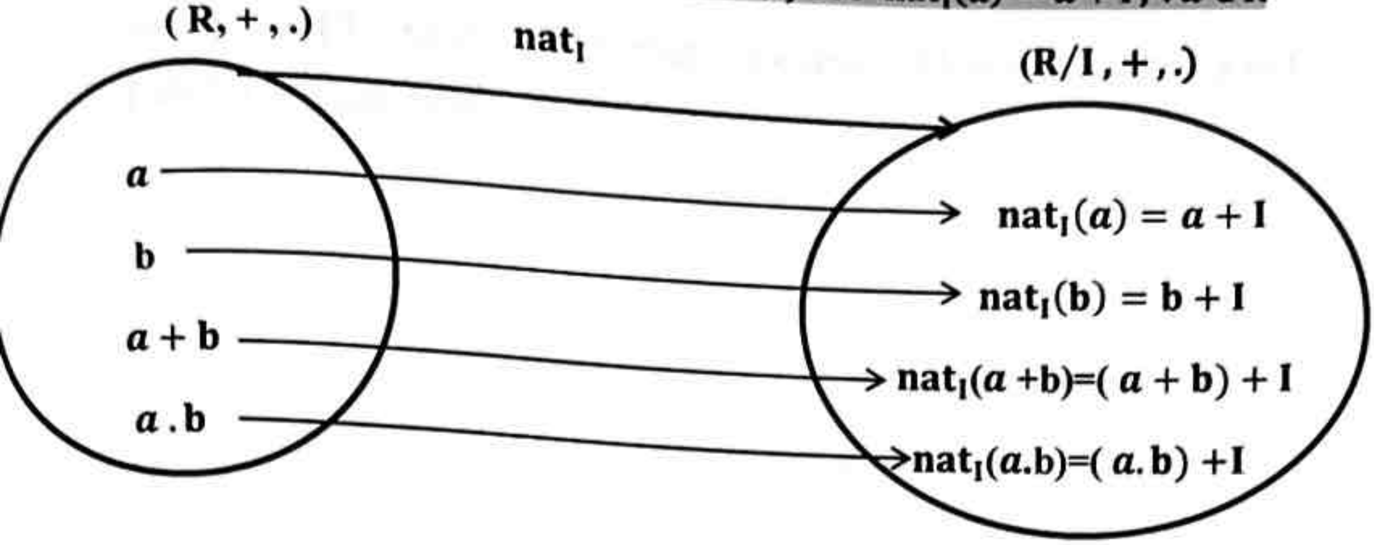
T.P. $nat_I : (R, +, .) \rightarrow (R/I, +, .)$ is

(i) ring homo

(ii) on to and

(iii) Ker. $(nat_I) = I$

(1) To prove : $nat_I : (R,+,.) \rightarrow (R/I,+,.)$ S.t $nat_I(a) = a + I, \forall a \in R$



Let a , $b \in R \Rightarrow nat_I(a) = a + I$ and $nat_I(b) = b + I$

Then

1- $nat_I(a + b) = (a + b) + I$
$= (a + I) + (b + I)$
$= nat_I(a) + nat_I(b)$

2- $nat_I(a . b) = (a . b) + I$
$= (a + I) . (b + I)$
$= nat_I(a) . nat_I(b)$

$\therefore$ nat_I is ring homomorphism

(ii) T-P , nat_I is on to .

$nat_I(R) = \{nat_I(a) : a \in R\}$

$= \{ a + I \ : a \in R \} = R/I$

$\therefore$ nat_I is on to .

(iii) To prove , Ker. $(nat_I) = I$.

Ker. $(nat_I) = \{ a \in R : nat_I \ (a) = \grave{0} \}$

$= \{ a \in R : a + I = 0 + I \}$

$= \{ a \in R : a + I = I \}$

$= \{ a \in R : a \in I \} = I$ [by Remark, $a + I = I$ iff $a \in I$]

Example : - Let $f:(Z,+,.)\rightarrow(Z/(3),+,.)$ be a function such that $f(a)=a+(3)$,$\forall a\in Z$. Show that f is epimorphism and find the Ker.f

(الحل مطابق للنظرية أعلاه)

The Fundamental Theorems of Ring Homomorphism

Theorem (1):- (First Fundamental Theorem) المبرهنة الاساسيه الاولى
Let f: (R, +, .) ⟶ (R̀, +̀ , .̀) be a ring homomorphism and onto function . Then (R/Kre. f , + ,.) ≃ (R̀, +̀ , .̀) .

Proof :-

المعطى f: (R, +, .) ⟶ (R̀, +̀ , .̀) be a ring homomorphism and onto

< To Prove , (R/Kre. f , + ,.) ≃ (R̀, +̀ , .̀) >?

Let g : (R/Kre. (f) , + ,.) ⟶ (R̀, +̀ , .̀) be a function defined by:

g(*a* + Ker. (f) = f(*a*) , ∀*a* + Ker. (f) ∈ R/Kre. (f) .

To prove g is isomorphism function , (i. e) to show

(1) g is homo
(2) g is onto
(3) g is one to one

<But , first we must to show g is well defined >?

Let a + Ker. (f) , b + Ker. (f) ∈ R/Kre. (f) such that

Let a + Ker. (f) = b + Ker. (f)

< To prove , g(b + Ker. (f))= g(*a* + Ker. (f)) >

∵ a + Ker. (f) = b + Ker. (f)

⟹ b−a ∈ Ker. (f) (by Remark , a +I = b+I iff b−a ∈I)

⟹ f(b−a) = 0̀ (by kernel definition)

⟹ f(b) − f(a) = 0̀ (since f is ring . homo)

⟹ f(b) = f(a)

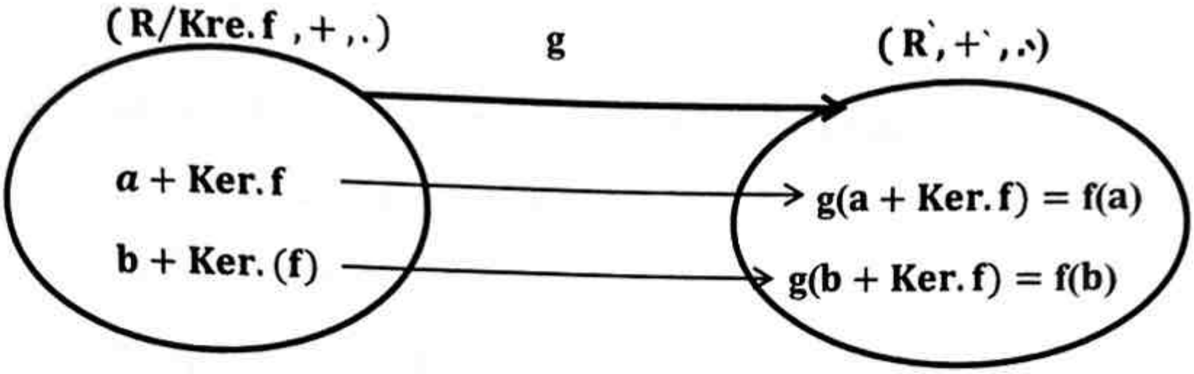
⟹ g(b + Ker. (f))= g(a + Ker. (f))

∴ g is well defined .

(1) < Now, To prove g is ring homomorphism>?

Let $g : (R/Kre.(f)\ ,+\ ,.) \to (R^{`}, +^{`}, .^{`})$ be a function defined by:

$g(a + Ker.(f) = f(a)\quad , \forall a + Ker.(f) \in R/Kre.(f)$.



Let $a + Ker.(f)\ ,\ b + Ker.(f) \in R/Kre.(f)\ ,\ a, b \in R$

$\therefore\ g(a + Ker.(f)) = f(a)$ and $g(b + Ker.(f)) = f(b)$

(I) $g((a + Ker.(f)) + (b + Ker.f)) = g((a + b) + Ker.(f))$

$= f(a + b)$ (بالتعويض بدالة g)

$= f(a) +^{`} f(b)$ (since f is ring. homo)

$= g(a + Ker.(f)) +^{`} g(b + Ker.(f))$

And

(II) $g((a + Ker.f) \cdot (b + Ker.f)) = g((a \cdot b) + Ker.f)$

$= f(a \cdot b)$ (بالتعويض بدالة g)

$= f(a) \cdot^{`} f(b)$ (since f is ring. homo)

$= g(a + Ker.f) \cdot^{`} g(b + Ker.f)$

$\therefore$ g is a ring . homo ⋯ (1)

(2) T.P. g is onto

$g(R/Kre.f) = \{g(a + Ker.f) : \forall a \in R\}$

$= \{f(a) : \forall a \in R\} = \grave{R}$.

$\therefore$ g is onto ⋯ (2)

(3) To show, g is 1-1

Let $a + Ker.f, b + Ker.f \in R/Kre.f$ such that

$g(a + Ker.f) = g(b + Ker.f)$

$\Rightarrow f(a) = f(b)$

$\Rightarrow f(b) - f(a) = \grave{0}$

$\Rightarrow f(b-a) = \grave{0}$

$\Rightarrow b-a \in Ker.f$ (by kernel definition)

$\Rightarrow a + Ker.f = b + Ker.f$ (by Remark, $a + I = b + I$ iff $b - a \in I$)

$\therefore$ f is 1-1 ⋯ (3)

Therefore, by (1), (2) and (3)

$\Rightarrow$ g is an isomorphism function $\Rightarrow$ By definition of ring isomorphic

$\therefore (R/Kre.f, +, .) \simeq (\grave{R}, \grave{+}, \grave{.})$. □