جامعة تكريت

كلية التربية بنات

قسم الرياضيات

تفاضل وتكامل

الاشتقاق

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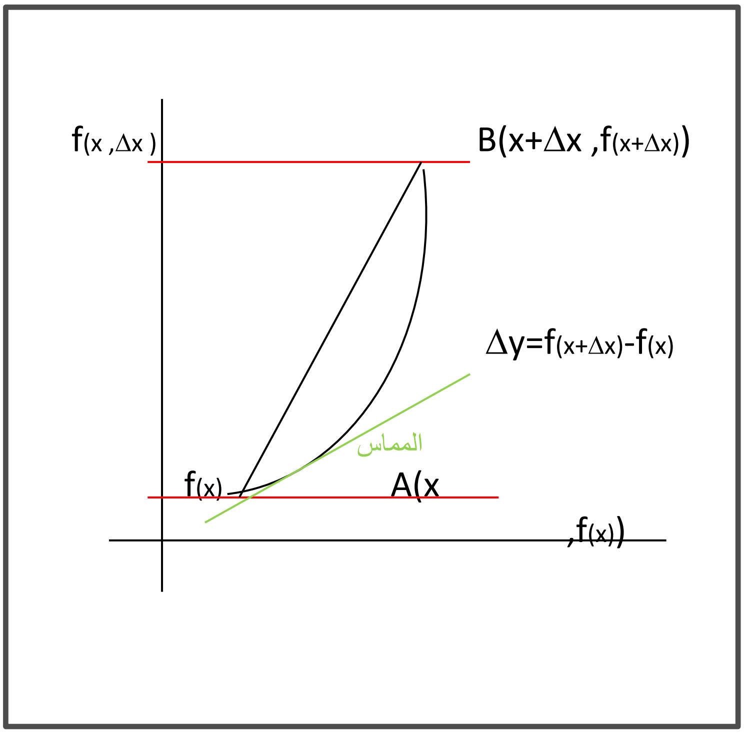
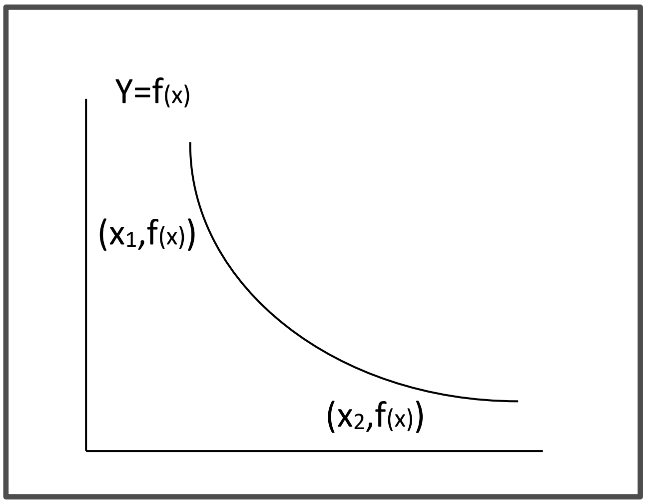
الاشتقاق Differentiation

لكل نقطة على بيان الدالة يوجد مستقيم مماس وحيد في تلك النقطة وميل المستقيم المماس لبيان الدالة f عند النقطة $(x, f_{(x)})$ يمثل المشتقة عند تلك النقطة.





لتكن $A(x, f_{(x)})$ نقطة ثابتة على المنحني , ولتكن $B(x+\Delta x, f_{(x+\Delta x)})$ هي نقطة أخرى عليه فأن :

$$\Delta y=f_{(x+\Delta x)}-f_{(x)}$$

$$m_{\text{sec}}=\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{x+\Delta x-x}=\frac{f(x+\Delta x)-(x)}{\Delta x}$$ ميل المستقيم القاطع

تلاحظ أنه عندما Δx يتناقص طولها (تقترب من الصفر)فان المستقيم القاطع (A B) يبدأ أكثر فأكثر بالانطلاق على المستقيم المماس عند النقطة $(x, f_{(x)})$ وهذا يعني أن ميل المستقيم القاطع (A B) سيكون مساويا لميل المستقيم المماس عند النقطة $(x, f_{(x)})$ وذلك عندما $(\Delta x \to 0)$ مع العلم أن ميل المستقيم المماس عند النقطة $(x, f_{(x)})$ يمثل مشتقة الدالة عند تلك النقطة .

والتمثيل الرياضي هو :

$$m_{\tan}=\lim_{\Delta x\to 0} m_{\text{sec}}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$$ ميل المماس

تعريف المشتقة → $\overline{f}(x) =$

$$\lim_{\Delta x\to 0}\frac{f(x=\Delta x)-f(x)}{\Delta x}$$

رموز المشتقة → $\overline{y}\ ,\ \overline{f}(x)\ ,\ \frac{dy}{dx}\ ,\frac{df(x)}{dx}$

$$\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$$

ملاحظة :

عندما تكون قيمة الغاية موجودة فأن الدالة تسمى دالة قابلة للاشتقاق (differentiable function) وf̅ تسمى مشتقتها عند النقطة x (the derivative off at x)

EX: let $f(x) = 4x - 2$,find $\overline{f}(x)$ by definition

Sol :

$$\overline{y} = \overline{f}(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$f(x) = 4x-2$

$f(x+\Delta x) = 4(x+\Delta x) - 2$

$$\overline{f}(x) = \lim_{\Delta x\to 0}\frac{4(x+\Delta x)-2-(4x-2)}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{4x+4\Delta x-2-4x+2}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{4\Delta x}{\Delta x} \qquad \Delta x\neq 0$$

$$= \lim_{\Delta x\to 0} 4 = 4$$

EX$_2$: let $f(x)=\sqrt{x}$,find the equation of the tangent line and normal line at the point (4,2) by definition .

Sol: $m_{\tan} = \bar{f}(x)$

$$\bar{f}(x) = \lim_{\Delta x\to 0}\frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\left(\frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}\cdot\frac{\sqrt{x+\Delta x}+\sqrt{x}}{\sqrt{x+\Delta x}+\sqrt{x}}\right)$$

$$=\lim_{\Delta x\to 0}\frac{x+\Delta x-x}{\Delta x(\sqrt{x+\Delta x}+\sqrt{x}}$$

$$= \lim_{\Delta x\to 0}\frac{\Delta x}{\Delta x(\sqrt{x+\Delta x}+\sqrt{x})}$$

$$= \lim_{\Delta x\to 0}\frac{1}{\sqrt{x+\Delta x}+\sqrt{x}}$$

$$= \frac{1}{\sqrt{x}+\sqrt{x}}$$

$$m_{\tan} = \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$(y - y_1) = m_{\tan}(x - x_1)$ معادلة المماس

$$y - 2 = \frac{1}{4}(x-4)$$

$$y = \frac{1}{4}x + 1$$

$m_{\perp} = \frac{-1}{mtan}$ ميل العمود

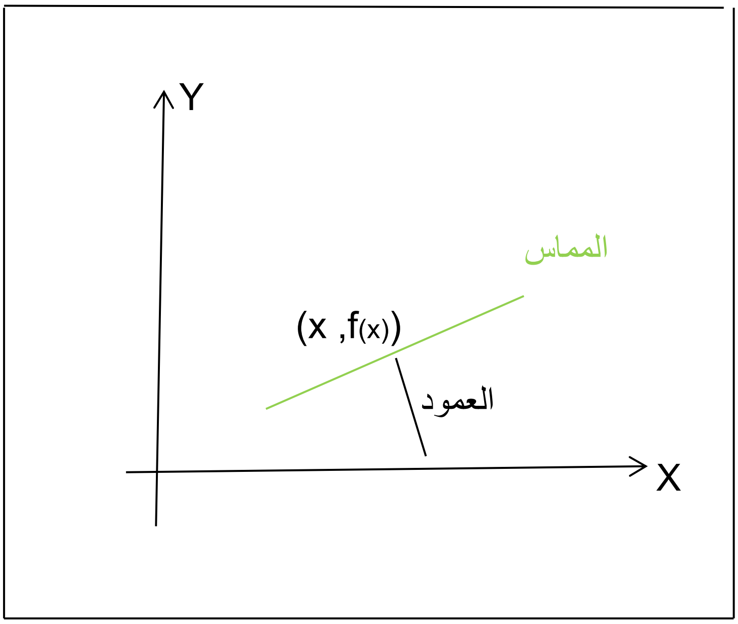
$$m_{\perp} = \frac{-1}{\frac{1}{4}} = -4$$

$(y - y_1) = m_{\perp}(x - x_1)$ معادلة العمود

$y - 2 = -4(x - x_1) \quad \rightarrow y = -4x + 18$

تعريف:

يعرف المستقيم العمود على منحني في نقطة معينه بأنه المستقيم العمود على المستقيم المماس عند نقطة التماس .



EX: find $\overline{f}(x)$ by definition :

1-$f(x)=x^3$

2-$f(x) = x^2 + \frac{1}{x}$

3- let $f(x) = x^2$,find the equation of the tangent line and normal line at the Point (3,9) by definition.

4-using definition to prove that $\overline{f}(x) = m$ for $f(x)=y=mx+b$.

5- find the tangent line at (6,3) for $y=\sqrt{x+3}$

Theorem : مبرهنة

Every function is differentiable at $x_\circ$,then f is continuous at $x_\circ$.

كل دالة قابلة للاشتقاق عند نقطة $x_\circ$ فأن f دالة مستمرة عند تلك النقطة .

Proof:

To prove $\text{Lim}_{x \to x_\circ} f_{(x)} = f_{(x_\circ)}$

$\text{Lim}_{x \to x_\circ} [f_{(x)} - f_{(x_\circ)}] = 0$

suppose that $\Delta x = x - x_\circ$

$x = x_\circ + \Delta x$

$f_{(x)} = f(x_\circ + \Delta x)$

when $x \to x_\circ$, then $\Delta x \to 0$

$\text{Lim}_{x \to x_\circ}[f_{(x)} - f_{(x)}] = \lim_{\Delta x \to 0}[f(x_\circ + \Delta x) - f(x_\circ)]$

$= \lim_{\Delta x \to 0}[\frac{f(x^\circ + \Delta x) - f(x^\circ)}{\Delta x} \cdot \Delta x]$

$= \lim_{\Delta x \to 0}\frac{f(x^\circ + \Delta x) - f(x^\circ)}{\Delta x} \cdot \lim_{\Delta x \to 0} \Delta x$

$= \overline{f}_{(x_\circ)} \cdot 0$

$= 0$

ملاحظة :

معكوس هذه المبرهنة غير صحيح ,اذا كانت الدالة f مستمرة عند النقطة $x_\circ$ فليس من الضروري أن تكون قابلة للاشتقاق عند تلك النقطة كما في المثال الاتي :

EX: let $f_{(x)} = |x|$, $x_\circ = 0$

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

$\overline{f}_{(x)} = \lim_{\Delta x \to 0}\frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$f(x) = \begin{cases} \Delta x \text{ if } \Delta x \geq 0 \\ -\Delta x \text{ if} \Delta x < 0 \end{cases}$$

$\overline{f}_{(x)} = \lim_{\Delta x \to 0}\frac{|x + \Delta x| - |x|}{\Delta x}$

$\overline{f}_{(x)} = \lim_{\Delta x \to 0}\frac{|0 + \Delta x| - |0|}{\Delta x}$

$= \lim_{\Delta x \to 0}\frac{|\Delta x|}{\Delta x} \longrightarrow L^+ = \lim_{\Delta x \to 0^+}\frac{\Delta x}{\Delta x} = 1$

$L^- = \lim_{\Delta x \to 0^-}\frac{-\Delta x}{\Delta x} = -1$ $\qquad L^+ \neq L^-$

Limit is not exist .

:· f is not differentiable function at $x_\circ=0$.

خواص المشتقات :

Theorem -1:· lit $f_{(x)}=c$,c is a constant , then $\overline{f}_{(x)}=0$.

Proof :· by definition

$$\overline{f}_{(x)}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{c-c}{\Delta x} \qquad ,f_{(x)}=c \quad ,f_{(x+\Delta x)}=c$$

$$=\lim_{\Delta x\to 0}\frac{0}{\Delta x}$$

$$=\lim_{\Delta x\to 0} 0 =0$$

EX: let $f_{(x)}=-5$, then $\overline{f}_{(x)}=0$

Theorem-2 :·lit f is a different table function at the point x ,and lit c is a constant , then (c. f)´is a differentiable function at x

$$(c\,.f)'_{(x)}=c.\,f'_{(x)}$$

proof:· by definition

$$(c\,.f)'_{(x)}=\lim_{\Delta x\to 0}\frac{(c\,.f)(x+\Delta x)-(c.f)(x)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{c.f(x+\Delta x)-c.f(x)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{c[f(x+\Delta x)-(c.f)(x)]}{\Delta x}$$

$$=c.\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$=c\,.f'_{(x)}$

$f'_{(x)}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$ لأنه بفرض f دالة قابلة للاشتقاق عند X

EX: lit $f_{(x)}=3x$, then $f'_{(x)}=3$, $\frac{dx}{dx}=1$

Theorem-3:·let $f_{(x)}$,$g_{(x)}$ are differentiable functions with respect to x ,then (f+ g) is differentiable function with respect to x ,

$(f+g)'_{(x)}=f'_{(x)}+g'_{(x)}$

Proof : by definition

$(f+g)'_{(x)}=\lim_{\Delta x\to 0}\frac{(f+g)(x+\Delta x)-(f+g)(x)}{\Delta x}$

$=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)+g(x+\Delta x)-(f(x)+g(x))}{\Delta x}$

$=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)+g(x+\Delta x)-f(x)-g(x)}{\Delta x}$

$=\lim_{\Delta x\to 0}\frac{\{f(x+\Delta x)-f(x)\}+\{g(x+\Delta x)-g(x)\}}{\Delta x}$

$=\lim_{\Delta x\to 0}(\frac{f(x+\Delta x)-f(x)}{\Delta x}+\frac{g(x+\Delta x)-g(x)}{\Delta x})$

$=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}+\lim_{\Delta x\to 0}\frac{g(x+\Delta x)-g(x)}{\Delta x}$

$=f'_{(x)}+g'_{(x)}$

لأنه بالفرض f ,g دوال قابلة للاشتقاق عند X

$f'_{(x)}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$

$g'_{(x)}=\lim_{\Delta x\to 0}\frac{g(x+\Delta x)-g(x)}{\Delta x}$

EX: lit $f_{(x)}=2x$,$g_{(x)}=1$

$(f+g)'_{(x)} = f'_{(x)} + g'_{(x)} = (2x)' + (1)' = 2(1)+0 = 2$

Remark : lit f_1 ,f_2 ,f_3 ,…,f_n are differentiable functions at x ,then

$(f_1 \pm f_2 \pm f_3 \ldots \pm f_n)'_{(x)} = f'_{1(x)} \pm f'_{2(x)} \pm f'_{3(x)} \pm \ldots \pm f'_{n(x)}$.

Theorem - 4 :let $y=f_{(x)}=X^{n}$, and let n be a positive integer number ,then

$$f'_{(x)} = \frac{d}{dx}(x^{n}) = nx^{n-1}$$

proof: By definition

$$f'_{(x)} = \mathrm{Lim}_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\frac{d}{dx}(x^{n}) = \mathrm{Lim}_{\Delta x \to 0} \frac{(x+\Delta x)^{n}}{x+\Delta x - x} - x^{n}$$

نصيف ونطرح x الى المقام

Let $x=b$,$x+\Delta x=a$

$$\frac{(x+\Delta x)^{n} - x^{n}}{(x+\Delta x) - x} = \frac{a^{n} - b^{n}}{a-b}$$

n من الحدود

$$= \frac{\cancel{(a-b)}(a^{n-1} + a^{n-2}b + a^{n-3}b^{2} + \cdots + ab^{n-2} + b^{n-1})}{\cancel{(a-b)}}$$

$$= (a^{n-1} + a^{n-2}b + a^{n-3}b^{2} + \cdots + ab^{n-2} + b^{n-1})$$

$$= (x+\Delta x)^{n-1} + (x+\Delta x)^{n-2}x + (x+\Delta x)^{n-3}.x^{2} + \ldots + (x+\Delta x).x^{n-2} + x^{n-1}$$

$$= \lim_{\Delta x \to 0}\{(x+\Delta x)^{n-1} + (x+\Delta x)^{n-2}.x + (x+\Delta x)^{n-3}.x^{2} + \ldots + (x+\Delta x).x^{n-2} + x^{n-1})\}$$

$$= \lim_{\Delta x \to 0}(x+\Delta x)^{n-1} + \lim_{\Delta x \to 0}(x+\Delta x)^{n-2}.\lim_{\Delta x \to 0} x$$

$$+ \lim_{\Delta x \to 0}(x+\Delta x)^{n-3}.\lim_{\Delta x \to 0} x^{2} + \ldots + \lim_{\Delta x \to 0}(x+\Delta x).\lim_{\Delta x \to 0} x^{n-2} + \lim_{\Delta x \to 0} x^{n-1}$$

$$= x^{n-1} + x^{n-2}.x + x^{n-3}.x^{2} + \ldots + x.x^{n-2} + x^{n-1}$$

$$= x^{n-1} + x^{n-1} + x^{n-1} + \ldots + x^{n-1} + x^{n-1} = nx^{n-1}$$

EX: let $f(x) = x^6$, find $\overline{f}(x)$

$\overline{f}_{(x)}=6x^5$

Theorem – 5 : let f(x) ,g(x) be two differentiable functions at x, then (f. g) is differentiable functions at x ,

$$(f.g)'_{(x)}=f_{(x)}.\overline{g}_{(x)}+g_{(x)}.\overline{f}_{(x)}$$

Proof: By definition

$$(f.g)'_{(x)}=\lim_{\Delta x\to 0}\frac{(f.g)(x+\Delta x)-(f.g)(x)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)\ .g(x+\Delta X)-f(x).g(x)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{f(x+\Delta x).g(x+\Delta x)-f(x).g(x)+f(x).g(x+\Delta x)-f(x).g(x+\Delta x)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{g(x+\Delta x)\{f(x+\Delta x)-f(x)\}+f(x)\{g(x+\Delta x)-g(x)\}}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\left\{g(x+\Delta x)\frac{f(x+\Delta x)-f(x)}{\Delta x}+f(x)\frac{g(x+\Delta x)-g(x)}{\Delta x}\right\}$$

$$=\lim_{\Delta x\to 0} g(x+\Delta x).\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}+\lim_{\Delta x\to 0} f(x)\ .\lim_{\Delta x\to 0}\frac{g(x+\Delta x)-g(x)}{\Delta x}$$

$$=g(x)\ .\overline{f}(x)+f(x).\overline{g}(x)$$

$$=f(x).\overline{g}(x)+g(x)\ .\overline{f}(x)$$

EX: let $f_{(x)}=(x^3+2)(1-x^2)$, find $\overline{f}_{(x)}$

$\overline{f}_{(x)}=(x^3+2).\frac{d}{dx}+(1-x^2)\frac{d}{dx}(x^3+2)$

$=(x^3+2).(-2x)+(1-x^2)(3x^2)$

$=-2x(x^3+2)+3x^2(1-x^2)$

Remark :let f, g, h are differentiable functions at x, then

$(f.\,g\,.h)'_{(x)}= f_{(x)}\,.(g.\,h)'_{(x)} +(g\,.h)_{(x)}\,.\overline{f}_{(x)}$

$= f_{(x)}\{\, g_{(x)}\,.\,.h'_{(x)} +h_{(x)}\,.g'_{(x)}\} +g_{(x)}.\, h_{(x)}.f'_{(x)}$

$=f_{(x)}\,.g_{(x)}\,.h'_{(x)} +f_{(x)}\,.h_{(x)}\,.g'_{(x)} +g_{(x)}\,.h_{(x)}\,.f'_{(x)}$

$=\overline{f}_{(x)}\,.g_{(x)}\,.h_{(x)} + g'_{(x)}\,.f_{(x)}.h_{(x)}+h'_{(x)}.f_{(x)}.g_{(x)}$

Theorem -6 :let $f_{(x)}$,$g_{(x)}$ two differentiable functions at x, if $g_{(x)}\neq 0$,then $(\frac{f}{g})$ is differentiable function at x : $(\frac{f}{g})'_{(x)} =\frac{g(x).f(x)-f'(x).\acute{g}(x)}{(g(x))^2}$

Proof : by definition

$$(\frac{f}{g})'_{(x)} = \lim_{\Delta x\to 0}\frac{\left(\frac{f}{g}\right)(x+\Delta x)-(\frac{f}{g})(x)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{\frac{f(x+\Delta x)}{g(x+\Delta x)}-\frac{f(x)}{g(x)}}{\Delta x}$$

$$= \lim_{\Delta x\to 0}(\{\frac{f(x+\Delta x)}{g(x+\Delta x)} - \frac{f(x)}{g(x)}\}.\frac{1}{\Delta x})$$

$$= \lim_{\Delta x\to 0} (\frac{g(x).f(x+\Delta x)-f(x).g(x+\Delta x)}{g(x).g(x+\Delta x)}\,.\frac{1}{\Delta x})$$

اضافة وطرح $F_{(x+\Delta x)}\,.g_{(x+\Delta x)}$ الى البسط

$$= \lim_{\Delta x\to 0} \frac{g(x).f(x+\Delta x)-f(x).g(x+\Delta x)+f(x+\Delta x).g(x+\Delta x)-f(x+\Delta x).g(x)}{\Delta x\,.g(x).g(x+\Delta x)}$$

$$= \lim_{\Delta x\to 0}\frac{-f(x+\Delta x)\{g(x+\Delta x)-g(x)\}+g(x+\Delta x)\{f(x+\Delta x)-f(x)\}}{\Delta x\,.g(x).g(x+\Delta x)}$$

$$=\lim_{\Delta x\to 0}\{\frac{-f(x+\Delta x)}{g(x).g(x+\Delta x)}.\frac{g(x+\Delta x)-g(x)}{\Delta x} +\frac{\cancel{g(x+\Delta x)}}{g(x).\cancel{g(x+\Delta x)}}\,.\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{-f(x+\Delta x)}{g(x).g(x+\Delta x)}\,.\lim_{\Delta x\to 0}\frac{g(x+\Delta x)-g(x)}{\Delta x} +\lim_{\Delta x\to 0}\frac{1}{g(x)}\,.\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$=\frac{-f(x)}{g(x).g(x)}\,.g'_{(x)} +\frac{1}{g(x)}\,.f'_{(x)}$ لأنه بالفرض f, g دوال قابلة للاشتقاق بالنسبة ل x

$$=\frac{-f(x).\acute{g}(x)}{(g(x))^2} +\frac{\acute{f}(x)}{g(x)}$$

$$=\frac{g(x).\acute{f}(x)-f(x).\acute{g}(x)}{(g(x))^2}$$

EX: lit $f_{(x)} = \frac{x+1}{x}$,find $f'_{(x)}$

$$f'_{(x)}= \frac{x.1-(x+1).1}{x^2} = \frac{x-x-1}{x^2} = \frac{-1}{x^2}$$

Corollary: نتيجة

Let $y=f_{(x)}=x^{-n}$,for $n\in Z_{1+}$, $x\neq 0$, then

$$y' = f'_{(x)} = \frac{dx^{-n}}{dx} = -nx^{-n-1}$$

proof : $\frac{dx^{-n}}{dx} = \frac{d}{dx}\left(\frac{1}{x^n}\right)$

$= \frac{x^n\frac{d(1)}{dx}-(1).\frac{dx^n}{dx}}{(x^n)^2}$ by – 6

$= \frac{x^n.0-n\,.\,x^{n-1}}{x^{2n}}$ by -1 & - 4

$= \frac{-n\,x^{n-1}}{x^{2n}}$

$= -nx^{-n-1}$

EX: let $f_{(x)} = -5x^{-3}$,find $f'_{(x)}$

$$f'_{(x)} = -5\,(-3x^{-4}) = 15x^{-4}$$

$\frac{d}{dx}(x^r) = r\,x^{r-1} \quad \forall\, r \in R$

Theorem -7 اشتقاق الدالة المركبة (قانون السلسلة)

Let g is a differentiable function at x ,f is a differentiable function at $g_{(x)}$,and let h=fog ,then h is a differentiable function at

$$h'_{(x)} = f'(g_{(x)})\,.g'_{(x)}$$

proof : by definition

$h'_{(x)}=\lim_{\Delta x\to 0}\frac{h(x+\Delta x)-h(x)}{\Delta x}$ $u=g_{(x)}$ ان:

$=\lim_{\Delta x\to 0}\frac{(fog)(x+\Delta x)-(fog)(x)}{\Delta x}$ $u+\Delta u=g_{(x+\Delta x)}$

$=\lim_{\Delta x\to 0}\frac{f\left(g(x+\Delta x)\right)-f\left(g(x)\right)}{\Delta x}$ $\Delta u=g(x+\Delta x)$

$=\lim_{\Delta x\to 0}\frac{f(u+\Delta u)-f(u)}{\Delta x}$ $\Delta x\to 0$ عندما

$=\lim_{\Delta x\to 0}\left(\frac{f(u+\Delta u)-f(u)}{\Delta x}.\frac{\Delta u}{\Delta u}\right)$ $\Delta u\to 0$ فان

$=\lim_{\Delta x\to 0}\frac{f(u+\Delta u)-f(u)}{\Delta u}.\lim_{\Delta x\to 0}\frac{\Delta u}{\Delta x}$ او $\frac{du}{dx}=\frac{dg(x)}{dx}=g'_{(x)}$

$=\lim_{\Delta u\to 0}\frac{f(u+\Delta u)-f(u)}{\Delta u}.\lim_{\Delta x\to 0}\frac{g(x+\Delta x)-g(x)}{\Delta x}$

$=f'_{(x)}.g'_{(x)}$ لان g دالة قابلة لاشتقاق عند x

$=f'(g_{(x)}).g'_{(x)}$

EX: lit $h_{(x)}=(2x^3+x^2-5x+1)^{15}$,find $h'_{(x)}$

sol:

let $g_{(x)}=2x^3+x^2-5x+1$,$f_{(x)}=x^{15}$

$h'_{(x)}=f'(g_{(x)}).g'_{(x)}$

$f'_{(x)}=15x^{14}$

$f'(g_{(x)})=15(g_{(x)})^{14}=15(2x^3+x^2-5x+1)^{14}$

$g'_{(x)}=6x^2+2x-5$

$h'_{(x)}=15(2x^3+x^2-5x+1)^{14}.(6x^2+2x-5)$

Corollary :

lit f is a differentiable function at x , and let $y=(f_{(x)})^n$, for $n\in z_1$,then $\frac{dy}{dx} = n(f_{(x)})^{n-1}.f'_{(x)}$

EXC: find $\frac{dy}{dx}$:

1. $y=(\frac{x^2+3}{x+1})^4$
2. $y=(2\sqrt{x}-1)^3$
3. $y=\sqrt{3-x^2}$
4. $y=\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}$
5. $y=(x^3+2)^2(1-x^2)^3$
6. $y=\frac{(1+2x^3)(1+x^4)}{x^2}$
7. $y=\sqrt{x+\sqrt{1+\sqrt{x}}}$
8. $y=x^3.\frac{1}{x^2+1}$
9. $y=\frac{\sqrt{x^2+1}}{(x+2)^4}$
10. $y=x^2(x^2+1)^{-\frac{1}{3}}$
11. let $f_{(x)}=x$,$g_{(x)}=x^2$, whets the value of x that makes the tangent line of two curves are parallel .
12. let $f_{(x)}=\frac{1}{\sqrt{x}}$,what the value of x that make the tangent of the curve when its parallel to line x+8y=10